Conversions between Binary to Octal and Hexadecimal (and vice-versa)

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Let us assume a n-bit number, N, in binary format. Let it be represented as follows:

$$N = x_n x_{n-1} \ldots x_1$$

Here, x_1 is the LSB (least significant bit) and x_n is the MSB (most significant bit). For simplicity let us assume that n is a multiple of 3 and 4. We have:

$$N = x_n 2^{n-1} + x_{n-1} 2^{n-2} + \ldots + x_2 2^1 + x_1 2^0 \tag{1}$$

Here, $x_1 \ldots x_n$ are binary digits. They can either be 0 or 1. We can subsequently write:

$$\begin{aligned} N &= x_n 2^{n-1} + x_{n-1} 2^{n-2} + \ldots + x_2 2^1 + x_1 2^0 \\ &= \underbrace{(x_n \times 2^2 + x_{n-1} \times 2^1 + x_{n-2} \times 2^0) 2^{n-3}}_{y_{n/3}} + \ldots + \underbrace{(x_6 \times 2^2 + x_5 \times 2^1 + x_4 \times 2^0) \times 2^3}_{y_2} + \\ &\quad \underbrace{(x_3 \times 2^2 + x_2 \times 2^1 + x_1 \times 2^0) \times 2^0}_{y_1} \\ &= y_{n/3} \times 8^{(n-3)/3} + \ldots + y_2 \times 8^1 + y_1 \times 8^0 \\ &= y_{n/3} \ldots y_2 y_1 \quad (\textit{in octal}) \end{aligned} \tag{2}$$

We thus have a method of converting a binary number into the octal (base 8) format by grouping bits in blocks of 3. We start from the LSB,

move leftward, group bits in a block of 3, and replace them by an octal digit.
Example:

Convert (110 001) in binary to base 8. Answer: 0 61

Example:

Convert (0 74) in base 8 to binary. Answer: 111 100

We can use the reverse technique to convert a number in base 8 to binary.

To convert a binary number to the hexadecimal format and vice-versa, we can follow the same logic and design a proof that says that we need to group bits starting from the LSB in groups of 4.
Example:

Convert (1100 0011) in binary to base 16. Answer: 0x C3

Example:

Convert (0x FE) in hex to binary. Answer: 1111 1110