Assorted Algorithms

Minimum Spanning Trees, Snapshots

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Outline

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1. Gallager Humblet Spira(GHS) Algorithm
   - Overview
   - Algorithms
   - Analysis

2. Distributed Snapshots
   - Chandy-Lamport Algorithm

Properties of an MST

Uniqueness

If each edge of the graph has a unique weight, then the MST is unique.

Construction based on Least Weight Edge

- A fragment is a sub tree of a MST.
- An outgoing edge of a fragment has one endpoint in the fragment, and one node outside the fragment.
- Proposition:

Theorem

If F is a fragment and e is the least weight outgoing edge, then $F \cup e$ is also a fragment.

GHS Overview

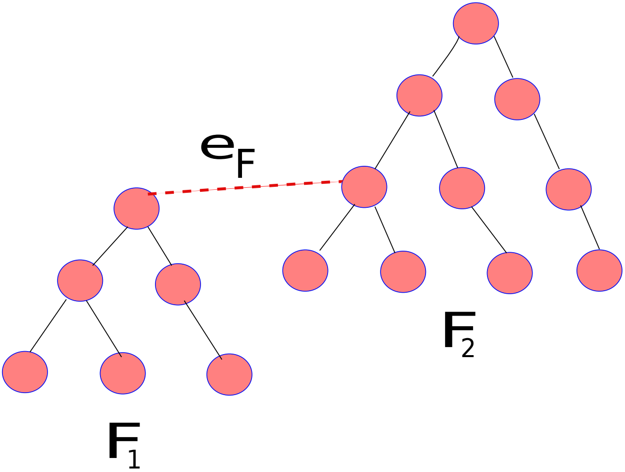
- Initially each node is a fragment.
- Gradually nodes fuse together to make larger fragments. A fragment joins another fragment by identifying its least weight outgoing edge.
- The nodes in a fragment run a distributed algorithm to co-operatively locate the least weight outgoing edge.
- Gradually the number of fragments decrease.
- Ultimately there is one fragment, which is the MST .

Properties of a Fragment

Properties of a Fragment

- Each fragment has a unique name.
- When two fragments combine, then all the nodes in one fragment change their name to a new name.
- Each fragment has a **level**.
  - Assume that F_1 is combining with F_2. It can only do so if $level(F_1) \leq level(F_2)$.
  - If $level(F_1) < level(F_2)$ then all the nodes in F_1 take on the name and level of F_2.
  - If $level(F_1) = level(F_2)$ then the level of both of the fragments gets incremented by 1.
    - The nodes of $F_1 \cup F_2$ get assigned a higher level (old level++).

Rules for Combining Fragments



Combining Rules

Let (F_1, L_1) be desirous of combining with (F_2, L_2). e_{F_1} is the least weight outgoing edge of F_1 and it terminates in F_2.

RULE *LT*

If $L_1 < L_2$, then we combine the fragments. All the nodes in the new fragment have name F_2 and level, L_2.

RULE *EQ*

If $L_1 = L_2$, and $e_{F_1} = e_{F_2}$. The two fragments combine, with all the nodes in the new fragment having:

- The level is $L_1 + 1$
- The name is e_{F_1}

RULE *WAIT*

Wait till any of the above rules apply.

Variables

state : sleep, find, found

- sleep The node is not initialized
- find The node is currently helping its fragment search for e_F.
- found e_F has been found

status [q] basic, branch, reject

- basic Edge is unused.
- branch Edge is a part of the MST.
- reject Edge is not a part of the MST.

name Name of the fragment.

level Level of the fragment

parent Points towards the combining edge.

bestWt,bestNode,rec,testNode temporary variables

Outline

Initialization

Current node *p*. Neighbor *q*.

Algorithm 1: Initialization

pq is the least weight edge from *p*
status[q] $\leftarrow$ branch
level $\leftarrow$ 0
state $\leftarrow$ found
rec $\leftarrow$ 0
send $<$**connect**,0$>$ to *q*

Process **connect** Message

Algorithm 2: Processing of the **connect** message

```
Receive <connect,L> from q:
if L < level then
    /* Combine with rule LT */
    status [q] ← branch
    send <initiate,level,name, state > to q
end
else if status [q] = basic then
    wait
end
else
    /* Combine with rule EQ */
    send <initiate,level+1,pq,find> to q
end
```

Receipt of **initiate** message

Algorithm 3: Processing of the **initiate** message

Receive <**initiate**,level',name',state'> from q:

/* Set the state */

(level,name, state) $\leftarrow$ (level',name',state')

parent $\leftarrow q$

/* Propagate the update */

bestNode $\leftarrow \phi$

bestWt $\leftarrow \infty$

testNode $\leftarrow$ *none*

foreach $r \in neigh(p)$: (*status* [r] = *branch*) $\wedge$ $(r \neq q)$ **do**

  send <**initiate**,level',name',state'> to r

end

/* Find least weight edge */

if *state* = *find* **then**

  rec $\leftarrow 0$

  findMin()

end

findMin

Algorithm 4: *findMin*

findMin:
if $\exists q \in$ *neigh(p): status [q] = basic, (w(pq) is minimal)* **then**
    testNode $\leftarrow$ q
    send $<$**test**,level,name$>$ to testNode
end
else
    testNode $\leftarrow \phi$
    report()
end

Receipt of **test** Message

Algorithm 5: Receipt of **test** Message

```
Receive <test,level',name'> from q
if level' > level then
    wait
end
else if name = name' then
    /* Internal Edge                                  */
    if status [q] = basic then
        status [q] ← reject
    end
    if q ≠ testNode then
        send <reject> to q
    end
    else
        findMin()
    end
end
else
    send <accept> to q
end
```

Receipt of **accept/reject** messages

Algorithm 6: Process accept/reject messages

Receive $<$**accept**$>$ from q:

testNode $\leftarrow \phi$

if $w(pq) < bestWt$ **then**

  bestWt $\leftarrow w(pq)$

  bestNode $\leftarrow q$

end

report()

Receive $<$**reject**$>$ from q:

if *status [q] = basic* **then**

  status [q] $\leftarrow$ reject

end

findMin()

report Method

Algorithm 7: *report* Method

report:

if $(rec = |\{q: status\ [q] = branch \wedge q \neq parent|\}) \wedge (testNode = \phi)$ **then**

  state $\leftarrow$ found

  send <**report**,bestWt> to parent

end

Receipt of **report** Message

Algorithm 8: Process *report* Message

```
Receive <report,ω> from q:
if q ≠ parent then
    if ω < bestWt then
        bestWt ← ω
        bestNode ← q
    end
    rec ← rec + 1
    report()
end
else
    if state ← find then
        wait
    end
    else if ω > bestWt then
        changeRoot()
    end
    else if ω = bestWt = ∞ then
        stop
```

changeRoot()

Algorithm 9: *changeRoot*() Method

```
changeRoot():
if status [bestNode] = branch then
    send changeroot to bestNode
end
else
    /* Along the Core Edge */
    status [bestNode] ← branch  send  <connect,level>  to
    bestNode
end
Receive changeroot:
changeRoot()
```

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Time Complexity

Proposition 1

There are $O(N\,log(N))$ fragment name or level changes.

Message Complexity

Message Complexity: 2E + 5Nlog(N)

- Every node is rejected only once $\rightarrow$ one **test** message and one **reject** message
  - Total: 2E messages
- At every level, a node sends/receives at most:
  1. receives: 1 initiate message
  2. receives: 1 accept message
  3. sends: 1 report message
  4. sends: 1 changeroot/connect message
  5. sends: 1 successful test message

Outline

Overview

- Every process can take a local snapshot.
- The process does not process any message while taking a snapshot

Consistent Snapshot

If a message receive event is part of a local snapshot, then its send event should also be part of a snapshot.

- The channels are FIFO

Algorithm

Algorithm 10: Chandy Lamport Algorithm

initialize:
take local snapshot
taken ← **true**
foreach $q \in$ *neigh(p)* **do**
    send <**mkr**> to q
end

Receive <**mkr**> :
if *taken* = **false then**
    take local snapshot
    taken ← **true**
    foreach $q \in$ *neigh(p)* **do**
        send <**mkr**> to q
    end

Analysis

Theorem 1

The algorithm terminates in finite time.

Theorem 2

If a message($p \rightarrow q$) is sent after a local snapshot, then it is not a part of the receiver's (q) snapshot.

Introduction to Distributed Algorithms by Gerard Tel, Cambridge University Press, 2000