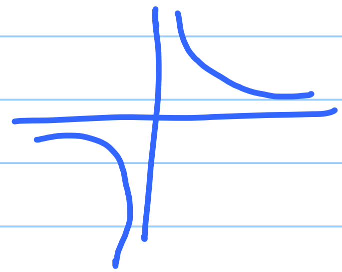
Sept-12

Note Title 12-09-2012

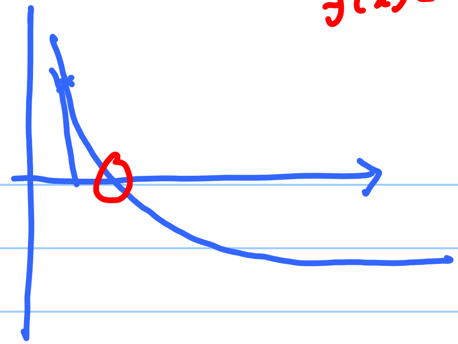
1) Floating point division.

Rectangular hyperbola



$$\frac{a}{b} = a \times \left(\frac{1}{b}\right)$$

$$f(x) = \frac{1}{x} - b$$



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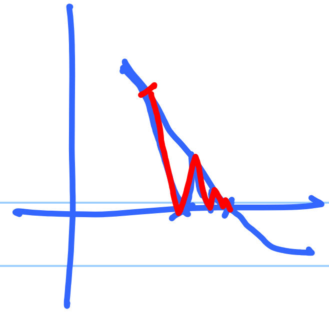

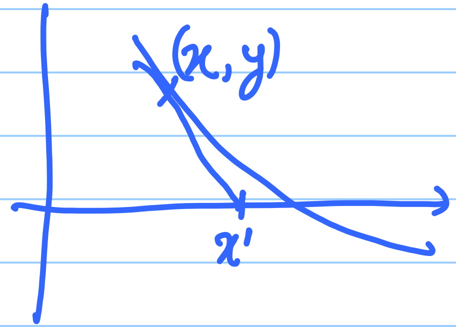

$$\frac{y-0}{x-x'} = f'(x) = \frac{-1}{x^2}$$

$$\Rightarrow -x^2 y = x - x'$$

$$\Rightarrow -x^2\left(\frac{1}{x} - b\right) = x - x'$$

$$\Rightarrow -x + bx^2 = x - x'$$

$$\Rightarrow x' = 2x - bx^2 \quad \{O(\log(n))\}$$ at the root

$\epsilon = 0$

$$\epsilon_x = b \cdot x - 1$$

$$\epsilon_{x'} = bx' - 1$$

$$= b(2x - bx^2) - 1$$

$$= 2bx - b^2x^2 - 1$$

$$= -(bx - 1)^2 = -\epsilon_x^2$$

$$\epsilon_{x'} = -\epsilon_x^2$$

This is a finite process.

$$\epsilon < 1$$

$$\Rightarrow bx - 1 < 1$$

$$\Rightarrow bx < 2$$

$$1 \leq b < 2$$

$$(x < 1)$$

IBM algorithm.

$$r = \frac{a}{b}$$

$$b = 1 - x$$

$$r = \frac{a}{(1-x)} \frac{(1+x)}{(1+x)} = \frac{a(1+x)}{1-x^2} = \frac{a(1+x)(1+x^2)}{(1-x^4)}$$

$$\ldots \ldots = \frac{a(1+x)(1+x^2)\cdots(1+x^{2^n})}{(1-x^{2^n})}$$

$$r = a(1+x)\cdots(1+x^{2^n})$$